

Qualifying Exam (Algebra I)

June 29, 2017

- 1.(10 points) Classify all groups of order 8 up to isomorphism.
- 2.(5 points each) Answer the following questions.
 - (a) Show that D_6 , the dihedral group of order 12, is a semi-direct product of two abelian groups.
 - (b) Is A_4 , the alternating group of degree 4, a semi-direct product of two abelian groups?
 - (c) Show that every nilpotent group is solvable.
 - (d) Give an example of a solvable group which is not nilpotent.
- 3.(5, 5, 10 points) Answer the following questions:
 - (a) Show that every free module is projective.
 - (b) Give an example of a projective module which is not free.
 - (c) Show that $\mathbb{Z}_m \otimes_{\mathbb{Z}} \mathbb{Z}_n \cong \mathbb{Z}_d$, where d is the greatest common divisor of m and n .
- 4.(5, 5, 10 points) Answer the following questions:
 - (a) Write down the class equation of S_5 .
 - (b) Write down the class equation of A_5 .
 - (c) Show that A_5 is simple.
5. (10 points) Let P be a Sylow p -subgroup of a finite group G . Show that $N_G(N_G(P)) = N_G(P)$.
- 6.(10 points) Determine whether the ring $\mathbb{Z}[\sqrt{-2}]$ is a Euclidean ring or not.
- 7.(10 points) Let R be the ring of all $n \times n$ matrices over a division ring D . Find all (two-sided) ideals of R .

The End