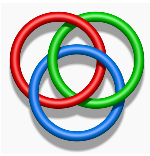
2018-1 QUALIFYING EXAM, ALGEBRAIC TOPOLOGY

Solve any 5 problems.

Problem 1 (20pts)**.** (1) (8pts) Describe a CW structure of $\mathbb{R}P^n$.
(2) (7pts) Using the above construction, compute all $H_n(\mathbb{R}P^n; \mathbb{Z})$.
(3) (7pts) Compute all $H_n(\mathbb{R}P^n; \mathbb{Z}_2)$.

Problem 2 (20pts)**.** Let L be the link complement shown below:



(1) (10pts) Compute all $H_n(X; \mathbb{Z})$ where $X = S^3 \backslash L$.
(2) (10pts) Compute all $H_n(Y; \mathbb{Z})$ where $Y = \mathbb{R}^3 \backslash L$.

Problem 3 (20pts)**.** Construct a 3-dimensional $\triangle$-complex X from 6-tetrahedra $T_1, \ldots, T_6$ by the following two steps. First arrange the tetrahedra in a cyclic pattern as in the figure below, so that each T_i shares a common vertical face with its two neighbors T_{i-1} and T_{i+1} (where subscripts being taken mod 6). Then identify the bottom face of T_i with the top face of T_{i+1} for each i.

(1) (15pts) Compute all $H_n(X; \mathbb{Z})$.
(2) (5pts) Compute all $H_n(X; \mathbb{Z}_6)$.

Problem 4 (20pts)**.** (1) (10pts) If $f : S^n \longrightarrow S^n$ has no fixed points, then the degree of f is $(-1)^n$.
(2) (10pts) If S^n has a continuous tangent vector field, then n is odd.

Problem 5 (20pts)**.** Let X be a finite dimensional manifold having a $\triangle$-complex structure. Show that

$$H_n^{\triangle}(X;\mathbb{Z}) \cong H_n(X;\mathbb{Z})$$

for all $n \geq 0$.

Problem 6 (20pts)**.** Let $i : D^k \longrightarrow S^n$ $(k < n)$ be an embedding. Show

$$\widetilde{H}_n(S^n - i(D^k);\mathbb{Z})) = 0$$

for all $n \geq 0$.