2016. 2학기 대수학 II 박사 자격시험

Show all the steps. Otherwise you may not get any points.

1. Prove that there are uncountably many automorphisms of C.

2. Let K be a Galois extension of F with finite cyclic Galois group G generated by $\sigma$.

   Prove that $\alpha \in$ K has the norm $N_{K/F}(\alpha)=1$

   if and only if $\alpha$ is of the form $\alpha=\frac{\beta}{\sigma(\beta)}$ for some nonzero $\beta \in$ K.

3. Let F≥K be a field extension.
   Prove that if u ∈ F is purely inseparable over K and v ∈ F is purely inseparable over K, then K(u,v)=K(u+v).

4. Prove that $Q(\sqrt{2}, \sqrt{3}, \ldots, \sqrt{n}) = Q(\sqrt{2} + \ldots + \sqrt{n})$ for all natural numbers n.

5. Let K be an algebraically closed field and R=$K[x_1, \cdots x_n, \cdots]$
   be a polynomial ring with denumerable number of indeterminates.
   Characterize the maximal ideals of R.

6. Let R be a finite boolean ring. Prove that R is isomorphic to the ring $Z_2 \oplus \cdots \oplus Z_2$.

7. Let F be the algebraic closure of Q.
   Prove that there is no subfield K of F such that [F:K]=3.

8. Let $\zeta_n \in C$ be a primitive nth root of unity. Let K:=$Q(\zeta_1, \ldots, \zeta_n, \ldots)$.
   Prove that $Aut_Q K$ is an abelian group, is not a cyclic group
   and its every element has infinite order.

9. Let F be the splitting field of Q over $f(x) = x^5 - 5$.
   Find the number of subfields of F.

10. Let K be a finite extension field of a field F with characteristic zero.
    Show that there exists an element u ∈ K such that K=F(u).