포항공과대학교 수학과

790-784 경북 포항시 남구 효자동 산31번지
Tel. 054-279-2712, Fax. 054-279-2799

POSTECH

Department of Mathematics
Pohang University of Science & Technology

San31 Hyoja-Dong Nam-Gu, Pohang 790-784, Korea
Tel.+82-54-279-2712, Fax. +82-54-279-2799

The Five-Gold-Button Math Competition

Problem 2008–12

Celebrating the end of the final exam of 2008 fall semester, Professor Hong at Department of Mathematics is organizing a blind date of n male students and n female students in his class. He wants students to find their partners using some math. Yes, he is a mathematician!

He prepared n blue cards with arithmetic formulas, for example, "$11 + 7 - 9$". The values of the formulas on the blue cards are all different. He also prepared n red cards with formulas with different values, in such a way that the value of a blue card is equal to that of exactly one red card. Each male student picks a blue card randomly, and each female student picks a red card randomly. Professor Hong expects conversations as follows, which he thinks romantic: "Hi, what is your formula?" "Oh, I have $11 + 7 - 9$." "Really? mine is $3 + 2 + 4$. What a perfect match!"

However, as you can guess, no student is a big fan of this way. They just want to find their partners as soon as possible and go out together, heading to a really romantic place. But it is not straightforward for all the participants to find partners. First of all, there are too many people, and furthermore, because Professor Hong wants only male–female conversations, it is not allowed to compare the values of blue cards, and similarly for red cards; the value of a blue card can be compared with only the value of a red card.

Therefore, you are asked to find an efficient way to match the partners. In other words, design an algorithm, prove that your algorithm always produces correct output, and analyze the efficiency. The gold button will be awarded to the most efficient correct algorithm.

To be more precise, the algorithm takes, as the input data, the values $b_1, \dots, b_n$, $r_1, \dots, r_n$ of the formulas on the blue and red cards. The algorithm can compare two numbers b_i and r_j; the result is either $=$, $<$, or $>$. In addition, the algorithm can choose a random index i, because in the real world one can pick a person randomly. The desired output can be in any reasonable form, for example, it could be a pair of permutations $\sigma, \tau \colon \{1, \dots, n\} \to \{1, \dots, n\}$ such that $b_{\sigma(i)} = r_{\tau(i)}$ for $i = 1, \dots, n$.

The efficiency is measured by the total number, say T, of comparisons performed during the execution of the algorithm. Because the input data and the random choices the algorithm makes may vary, we want to minimize the expected value $E(T)$, viewed as a function of n. To simplify the problem, we are interested only in the asymptotic order of $E(T)$; for example $E(T) = 3n^2$ and $E(T) = n^2 + 5n$ are regarded as equivalent efficiency because both are of order n^2. (If you are familiar with notation like $O(n^2)$ or $\Theta(n^2)$, then it is exactly what we are interested in.)

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Correspondence: `five-gold@postech.ac.kr`