

1. (10) Let  $X_1$  and  $X_2$  be two independent random variables with joint density  $f(x_1, x_2) = 4x_1x_2$  for  $0 < x_1 < 1, 0 < x_2 < 1$ .
  - (a) (6) Find the probability  $P(|X_1 - X_2| \leq y)$  for  $0 < y < 1$ .
  - (b) (4) Obtain  $E(|X_1 - X_2|)$ .
  
2. (12) Let  $X_1, X_2, \dots, X_n$  be independent random variables from a distribution with mean  $\mu$  and variance  $\sigma^2$ .
  - (a) (5) Obtain the mean and variance of  $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ .
  - (b) (7) Show that the mean of  $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$  is  $\sigma^2$ .
  
3. (8) Suppose that a survey showed that 80 of 100 randomly chosen Postech students said that they are unsatisfied with the cafeteria meals. Find a 95% confidence interval for the true proportion of unsatisfied Postech students.  
(For the standard normal random variable  $Z$ , the points  $z_\alpha$  satisfying  $P(Z > z_\alpha) = \alpha$  are (i) 1.645 for  $\alpha = 0.05$ , (ii) 1.96 for  $\alpha = 0.025$ , and (iii) 2.24 for  $\alpha = 0.0125$ .)