**MATH 351**
**Introduction to Numerical Analysis** **November 2016**

**1.** A clamped cubic spline $s$ for a function $f$ is given on $[1,3]$ by

$$s(x) = \begin{cases} 3(x-1) + 2(x-1)^2 - (x-1)^3, & \text{if } 1 \le x < 2, \\ a + b(x-2) + c(x-2)^2 + d(X-2)^3, & \text{if } 2 \le x \le 3. \end{cases}$$

Given $f'(1) = f'(3)$, find $a, b, c$, and $d$.

**2.** (a) Find the Legendre polynomials $L_0(x), L_1(x), L_2(x), L_3(x)$ which are mutually orthogonal with respect to the inner product $(f,g) = \int_{-1}^{1} f(x)g(x)\,dx$.

(b) Let $Q_3(f) = w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3)$ be the Gaussian quadrature rule to approximate $\int_{-1}^{1} f(x)dx$, where $x_1, x_2$, and $x_3$ are the roots of $L_3(x)$.

Find $x_1, x_2, x_3$ and $w_1, w_2, w_3$.

(c) Find the degree of precision (accuracy) $d$ of $Q_3(f)$.

**3.** Let

$$A = \begin{bmatrix} 0 & 1 & 3 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}.$$

By using the partial pivoting of $A$, we can have

$$PA = LU,$$

where $P$ is a permutation matrix, $L$ is a lower triangular matrix with diagonals 1, and $U$ is an upper triangular matrix. Find these matrices $P, L$, and $U$.

The End