

Introduction to Differential Geometry

Graduation Qualification Exam

November 1, 2016

[1] Let α be the plane-curve defined by

$$\alpha(t) = ((2 + 3 \cos t) \cos t, (2 + 3 \cos t) \sin t),$$

where $0 \leq t \leq 2\pi$. Find all the vertices (i.e., the points on the curve where the curvature takes local extrema) of this curve.

[2] Assume that the space curve α is regular (i.e., the speed is nowhere zero) and satisfies the condition $|\alpha(t)| = 2$ for any t . Show that the curvature of α is nowhere zero.

[3] A surface M in \mathbb{R}^3 is parametrized by

$$\mathbf{x}(u, v) = (uv, v^2, u^2),$$

where $u > 0$, $v > 0$. Find an explicit expression of its second fundamental form in terms of (u, v) -coordinates.