1. (Counting) We can fill $n \times n$ deficient board (with only one square empty) with trominoes for $n = 2^k$ ($k \in \mathbb{N}$) using the following algorithm.

```
tile(n, L){\\n is the size of the board and  \\L is the location of empty square
      if(n==2) {tile with a tromino; return success;}
      divide the board into four (n/2)X(n/2)-boards;
      rotate the board so that L is located in the upper left quadrant;
      locate a tromino in the center of the board;
      tile(n/2,m_1);
      tile(n/2,m_2);
      tile(n/2,m_3);   //m_1,m_2,m_3,m_4 are locations of empty squares
      tile(n/2,m_4);   //in each smaller board
}
```

((a), (b) 둘중 하나이상 풀면 10점)

(a) Using the idea of this algorithm, prove that

$$2^{2k} - 1 = 3 \cdot 2^{2(k-1)} + 3 \cdot 2^{2(k-2)} + \cdots + 3 \cdot 2^2 + 3.$$

(b) Using a three dimensional analogy of this algorithm of filling a region of size $2^k \times 2^k \times 2^k$ (with only one cube empty) with **septo-block**($2 \times 2 \times 2$-block with one cube missing) and prove that

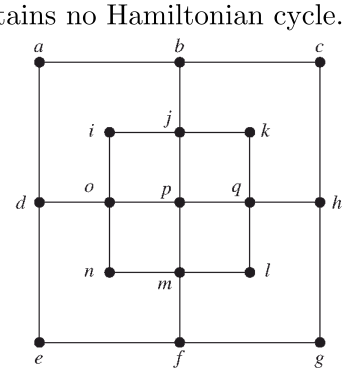
$$2^{3k} - 1 = 7 \cdot 2^{3(k-1)} + 7 \cdot 2^{3(k-2)} + \cdots + 7 \cdot 2^3 + 7.$$

2. ((a), (b) 둘중 하나이상 풀면 10점) Prove (a) or (b).

(a) Prove that Fibonacci sequence defined by $f_1 = f_2 = 1, f_n = f_{n-1} + f_{n-2},\ \ n \geq 3$ satisfies:

$$f_{n+1} f_{2n} + f_n f_{2n-1} = f_n f_{2n+1} + f_{n-1} f_{2n}, \quad (n \geq 2).$$

(b) Show that the following graph contains no Hamiltonian cycle.



3. (Hall's marriage Theorem : 10점) Let A be an $n \times n$ matrix whose entries are all nonnegative and satisfy

$$\sum_{i=1}^{n} a_{ij} = \sum_{j=1}^{n} a_{ij} = 1.$$

Sometimes we call this matrix A a **bistochatic matrix**. Now, show that we can choose n entries $\{a_{i_k j_k} > 0 | 1 \leq k \leq n\}$ of A so that $i_k \neq i_\ell$ and $j_k \neq j_\ell$ if $k \neq \ell$. (In other words, $\{i_k : 1 \leq k \leq n\} = \{j_k : 1 \leq k \leq n\} = \{1, 2, \cdots, n\}$.)

Festina Lente! (천천히 서두르라!)