

Calculus Graduation Exam

- (a) State Stokes' theorem .
(b) Let \mathbf{F} be a vector field given by $\mathbf{F}(x, y, z) = (-y, x, z)$. Let S be the part of the paraboloid $z = 7 - x^2 - 4y^2$ that lies above the plane $z = 3$, oriented with upward pointing normal. Use Stokes' Theorem to find

$$\iint_S \operatorname{curl} \vec{F} \bullet d\vec{S}$$

- Evaluate the following limits

(a)

$$\lim_{x \rightarrow -1} \frac{x^2 - 25}{x^2 - 4x - 5}$$

(b)

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{|x - 1|}$$

- Find all critical points and their types (local min, local max or saddle points) of the function on \mathbb{R}^2 :

$$F(x, y) = \frac{1}{2}x^2 + y^3 - 3xy - 4x + 2.$$