Graduation Exam, Fall 2016
(Analysis)

1.Let f and g be continuous mappings of a metric space X into a metric space Y, and let E be a dense subset of X. If $g \equiv f$ in E, prove that $g \equiv f$ in X.

2. Find a function f on $[0, 1]$ such that f is not Riemann integrable on $[0, 1]$, but f^2 is Riemann integrable on $[0, 1]$.

3. For any two real sequences $\{a_n\}$, $\{b_n\}$, prove that

$$\limsup_{n\to\infty}(a_n + b_n) \leq \limsup_{n\to\infty} a_n + \limsup_{n\to\infty} b_n$$

provided that the sum on the right is not of the form $\infty - \infty$.