

2016–1 Qualifying Exam

Algebraic Topology

1. (10pts) Prove or disprove: every map $S^1 \times S^1 \rightarrow S^2$ is null homotopic.
2. (15pts) Show that any map $f: S^n \rightarrow S^n$ with $\deg f \neq (-1)^{n+1}$ has a fixed point.
3. Let $T^n = S^1 \times \cdots \times S^1$ be the n -torus.
 - (a) (10pts) Describe a CW complex structure on T^n .
 - (b) (10pts) Compute $H_*(T^n)$ using the cellular chain complex.
 - (c) (10pts) Find a space X which is not homotopy equivalent to T^n but satisfies $H_i(T^n) \cong H_i(X)$ for any i .
4. (15pts) Show that there is a non-vanishing tangent vector field on S^n if and only if n is odd.
5.
 - (a) (10pts) Suppose $X_1 \subset X_2 \subset \cdots$ are open subsets of a space X such that $X = \bigcup_i X_i$. Show $H_k(X) \cong \varinjlim_i H_k(X_i)$ for any k .
 - (b) (10pts) Show that if $f: S^1 \rightarrow S^2$ is an embedding, then $S^2 - f(S^1)$ has two components.
 - (c) (10pts) If X and $\{X_i\}$ are as in (a), is $H^k(X)$ isomorphic to $\varinjlim_i H^k(X_i)$? Give a reason for your conclusion.