

1. (10pts) What is the probability that if n people randomly reach into a dark closet to retrieve their hats, no people will pick their own hat? Here, obtain the probability by taking the limit on the fraction of outcomes in which no person gets their own hat.

2. (5pts) We have 15 personal computers and 10 printers. Every five minutes, some subset of the computers requests printers. At least how many different connections between various computers and printers are necessary to guarantee that if at most 10 computers want a printer, there will always be connections to permit each of these computers to use a different printer?

3. (5pts) Solve the recurrence relation $a_n = 3a_{n-1} - 4n + 3 \cdot 2^n$ with initial condition $a_1 = 8$.

4. (5pts) Let G be a graph with n edges and m vertices, v_1, v_2, \dots, v_m . Then show that $\sum_{i=1}^m \delta(v_i) = 2n$, where $\delta(v_i)$ is the degree of a vertex v_i .

5. (5pts) Let G be a connected planar graph with n edges, and v vertices, and f faces. Using mathematical induction on the number of edges, show that $v - e + f = 2$ for the case that G contains no cycles.