1. (배점 : 10점) Let $f(z)$ be a doubly periodic **entire** function satisfying

$$f(z+2\pi) = f(z+2\pi i) = f(z), \quad \text{for all} \quad z \in \mathbb{C}.$$

Then show that $f(z)$ is a constant function.

2. (배점 : 하나풀면 7점, 둘다풀면 10점) Calculate the following integrals using residue :

(a) $\displaystyle\int_0^\infty \frac{1}{(x^2+a^2)(x^2+b^2)}\,dx$, where $a > b > 0$,

(b) $\displaystyle\int_0^{2\pi} \frac{1}{5+4\sin\theta}\,d\theta$. (Hint : Recall that $\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$.)

3. (배점 : 10점) Let $f : A \subset \mathbb{C} \to \mathbb{C}$ be analytic on a region $\mathcal{R} \subset A$ with $f'(z) \neq 0$ for any $z \in \mathcal{R}$. Let C_1, C_2 be two smooth curves in $\mathcal{R}$ which intersect at $\alpha \in \mathcal{R}$. Let ψ be the acute angle between the tangent lines to C_1 and C_2 at α. Let $\widetilde{C}_1 = f(C_1)$ and $\widetilde{C}_2 = f(C_2)$ be the image curves in codomain intersecting at $\beta = f(\alpha)$.

Now, show that the acute angle between tangent lines to $\widetilde{C}_1$ and $\widetilde{C}_2$ at β is also ψ.