2016-1학기 학부 해석학 졸업시험

1. Suppose $f(x) \coloneqq \sum_{n=0}^{\infty} a_n (x-a)^n$, $|x-a| < R$. Show that

$$f'(x) = \sum_{n=1}^{\infty} n a_n (x-a)^{n-1}.$$

2.1. Let E be an open set in R^n. Suppose f maps E into R^m, and $x \in E$. Define $f'(x), \|f'(x)\|$.

2.2. Suppose f is a continuous map of $[a,b]$ into R^k and f is differentiable in (a,b). Show that there exists $x \in (a,b)$ such that

$$|f(b) - f(a)| \leq (b-a)|f'(x)|$$

(Hint: Put $\phi(t) := \big(f(b) - f(a)\big) \cdot f(t)$, $a \leq t \leq b$.)

3. Let α be a monotonically increasing function on $[a,b]$. Suppose f is continuous on $[a,b]$. Show that the Riemann- Stieltjes integral $\int_a^b f d\alpha$ exists.