1. Show that if the events $B_1, B_2, \cdots, B_k$ constitutes a partition of the sample space S such that $P(B_i) \neq 0$ for $i = 1, 2, \cdots, k$, then for any event A in S such that $P(A) \neq 0$,

$$P(B_r|A) = \frac{(B_r \cap A)}{\sum_{i=1}^{k} P(B_i \cap A)} = \frac{P(B_r)P(A|B_r)}{\sum_{i=1}^{k} P(B_i)P(A|B_i)} \quad \text{for } r = 1, 2, \cdots, k.$$

2. Let the random variable X have a gamma distribution with parameters α and β. Then its density function is given by

$$f(x; \alpha, \beta) = \begin{cases} \dfrac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} & \text{for } x > 0, \\ 0 & \text{elsewhere,} \end{cases}$$

where $\alpha > 0$ and $\beta > 0$. Derive the moment generating function $M_X(t)$ of the gamma random variable X and then use it to find the mean value μ and the variance σ^2 of the gamma random variable X.

3. The average zinc concentration recovered from a sample of measurements taken in 36 different locations in a river is found to be 2.6 grams per milliliter. Find the 95% confidence interval for the mean zinc concentration in the river. Here, assume that the population standard deviation is 0.3 gram per milliliter and use the $z-$value $z_{0.025} = 1.96$.