1. Let a function $f(x) \in C^2[a, b]$, and suppose that $f(p) = 0$ and $f'(p) \neq 0$ for $p \in (a, b)$. Then show that Newton's Method generates a sequence $\{p_n\}_{n=0}^{\infty}$ converging to p quadratically. Here we assume that a initial approximation p_0 is chosen properly in the neighborhood of $p \in (a, b)$.

2. Let $f(x) \in C^2[x_0, x_1]$ and let $P(x)$ be the linear Lagrange interpolating polynomial for $f(x)$, i.e.,
$$P(x) = f(x_0)L_0(x) + f(x_1)L_1(x) = f(x_0)\frac{x - x_1}{x_0 - x_1} + f(x_1)\frac{x - x_0}{x_1 - x_0}.$$
Then show that for each $x \in [x_0, x_1]$, there exists a number $\xi(x) \in (x_0, x_1)$ such that $f(x) = P(x) + \frac{f''(\xi(x))}{2!}(x - x_0)(x - x_1)$.

3. Let $f(x) \in C^2[a, b]$ and $h = b - a$. Show that the Trapezoidal rule for approximating the integral is given by
$$\int_a^b f(x)\,dx = \frac{h}{2}[\, f(a) + f(b)\,] - \frac{h^3}{12}f''(\xi) \quad \text{for some } \xi \in (a, b).$$