

Introduction to Differential Geometry

1. Consider the parametrized surface given below:

$$\mathbf{x}(u, v) = ((c + a \cos(v)) \cos(u), (c + a \cos(v)) \sin(u), z \sin(v)), (u, v) \in \mathbb{T} \times \mathbb{T},$$

where \mathbb{T} is the interval $[0, 2\pi]$ with 0 identified with 2π .

- i) Compute the first fundamental form.
 - ii) Describe the regions of the surface where the Gaussian curvatures are respectively positive, zero, and negative.
2. Compute the torsion and curvature of the curve below.

$$\gamma(t) = (3 \cos(t), 3 \sin(t), 5t)$$

3. Let γ be a curve on a surface S . Show that if γ is both a geodesic and an asymptotic curve then it is a plane curve.