1. **(10pts):** Find the sum of the two hexadecimal numbers m and n given by

$$m = 4F53_{16}, \quad n = 12C7_{16}.$$

(The result should be in hexadecimal form.)

2. Consider the hyper 4-cube H_4.

(a) **(5pts):** A graph $G = (V, E)$ is said to be bipartite if there is a bi-partition(a partition composed of two subsets) $\{V_1, V_2\}$ of V such that $V = V_1 \cup V_2$ with $V_1 \cap V_2 = \emptyset$ and each edge $e \in E$ is incident on one vertex from V_1 and on one vertex from V_2.
Show that H_4 is a bipartite graph. (Hint : Consider the number of 1's in the coordinate of each vertex.)

(b) **(5pts):** find a Hamiltonian cycle in H_4 (i.e, a 4-bit Gray code) in other words a simple cycle in H_4 which visits every vertex.

3. We define the incidence matrix $M = [m_{ve}]_{v\in V, e\in E}$ of a graph $G = (V, E)$ with each vertex as a row index and each edge as a column index, where $m_{ve} = 1$ if e is incident on v, $m_{ve} = 2$ if e forms a loop from v to v and otherwise $m_{ve} = 0$. Using the fact

$$\sum_{v\in V}\sum_{e\in E} m_{ve} = \sum_{e\in E}\sum_{v\in V} m_{ve},$$

we immediately see that $\sum_{v\in V} \delta(v) = 2|E|$, where $\delta(v) = \sum_{e\in E} m_{ve}$ is called the degree of the vertex v. Let's call this fact the conservation law. **Immitating this conservation idea, prove the following** :

(a) **(5pts):** In a soccer ball structure(C_{60}:Fullerene), find the number e of edges, USING the fact that there are twelve pentagons and twenty hexagons in a fullerene and at every vertex one pentagon and two hexagons meet.
(Hint :You may define the incidence matrix $M^{(EF)}$ between the set of edges and the set of faces.)

(b) **(5pts):** Suppose that a polyhedron(다면체) is composed of n_1 pentagons and n_2 triangles with $n_1, n_2 \in \mathbb{N}$. Show that both n_1 and n_2 are even or both are odd.

Good Luck!