Exam for Graduation

[November 2015-Applied Complex Variables]

1. Let $f(z) = \bar{z}^3$.

   (a) Show that $f'(z)$ does not exists at any nonzero point.

   (b) Show that $f'(0)$ exists and then find the value.

2. Evaluate $\displaystyle\int_C \frac{\cos z}{z^2(z^2+4)}\,dz$, where C is the positively oriented circle $|z| = 3$.

3. Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{x \sin x}{(x^2+1)^2}\,dx$.