

Analysis 1

1. Show that continuous function $f : [0,1] \rightarrow \mathbb{R}$ is uniformly continuous.
2. Show that half open interval $[0,1)$ is not compact in \mathbb{R} .
3. Let f be a continuously differentiable function. Suppose that

$$\{x: f(x) \neq 0\} \subset (-N, N) \text{ for some } N > 0.$$

Show that

$$\lim_{n \rightarrow \infty} \int_{-\infty}^{\infty} f(x) \sin nx \, dx = 0.$$