

1. Let X be a nonnegative random variable with mean $\mu < \infty$. Show that for any $k > 0$,

$$P(X \geq k\mu) \leq \frac{1}{k}.$$

2. Let X_1 and X_2 be two independent unbiased estimators for θ with (known) variances 1 and 2, respectively. Consider a combined estimator of the form $Y_a = aX_1 + (1 - a)X_2$ for some real a .

(a) Show that Y_a is unbiased for θ .

(b) Choose a value of a at which the variance of Y_a is the smallest.

3. Suppose that a survey showed that 80 of 100 randomly chosen Postech students said that they are unsatisfied with the cafeteria meals. Find a 95% confidence interval for the true proportion of unsatisfied Postech students.
(You may use tables on the next page.)

TABLE A1 Standard Normal Distribution Function: $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} dt$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
10	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
11	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
12	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
13	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
14	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
15	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
16	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
17	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
18	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
19	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
20	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
21	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
22	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
23	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
24	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
25	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
26	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
27	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
28	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
29	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
30	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
31	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
32	.9993	.9993	.9994	.9994	.9994	.9994	.9995	.9995	.9995	.9995
33	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
34	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

TABLE A3 Values of α_n

n	$\alpha = .10$	$\alpha = .05$	$\alpha = .025$	$\alpha = .01$	$\alpha = .005$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.474	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
∞	1.282	1.645	1.960	2.326	2.576