Math 351 Introduction to Numerical Analysis

Graduation Exam
May, 2015

1. (10 points) We want to find a sequence $\{x_n\}$ which converges to $\sqrt{5}$.

(a) Find a function $f(x)$ such that $f(\sqrt{5}) = 0$ and find the formula between x_n and x_{n+1}, where the terms x_n are generated by Newton's method.

(b) Evaluate the term x_2 with the initial guess $x_1 = 1$.

2. (10 points) Let $f(x) = \sin 2x$ on the interval $[0, \pi/2]$. For a positive integer n, let $x_j = (\frac{\pi}{2n})j, j = 0, 1, \ldots, n$. Let $S_n(x)$ be the piecewise linear polynomial such that $S_n(x_j) = f(x_j), \quad j = 0, 1, \ldots, n$.

For the error bound

$$\max_{0 \le x \le \pi/2} |S_n(x) - f(x)| \le 10^{-8},$$

find an inequality of n to get the smallest n.

3. (10 points) Let

$$A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 5 \end{bmatrix}.$$

(a) Find the matrices L and U such that $A = LU$, where the matrix L is a lower triangular matrix with diagonals 1's and U is an upper triangular matrix.

(b) Find the diagonal matrix D such that $A = LDL^T$, where L^T is the transpose matrix of L.

(c) By finding the sum of complete squares of the following quadratic form of A, show that the matrix A is positive definite.

$$\begin{array}{l} x_1(4x_1 + 2x_2 + 2x_3) \\ +x_2(2x_1 + 6x_2 + 2x_3) \\ +x_3(2x_1 + 2x_2 + 5x_3) \end{array} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

The End