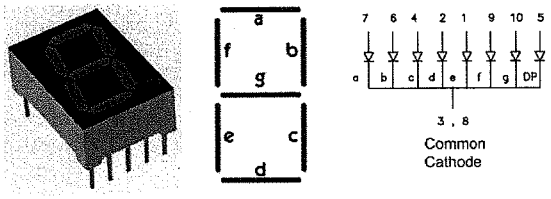
1. Let $f(x) = x^2$ and $\alpha = \sum_{n=1}^{\infty} \frac{x_n}{2^n}$ with $x_n \in \{0, 1\}$ for all $n \in \mathbb{N}$.

   (a) **(5pts):** Find the binary representation of e upto fifth bit under the binary floating point, in other words, write in the form of $(a_1 a_2 . b_1 b_2 b_3 b_4 b_5)_2$.
   (Hint : Use $e = 2.71828182845904523536028 7 \cdots$.)

   (b) **(5pts):** Suppose that
   $$f \circ f(e^{i2\pi e}) = e^{i2\pi\alpha}.$$
   Now, find x_k for $1 \leq k \leq 3$. (Hint : Use the result of (a).)

2. **(10pts):** Represent the following expression as a binary tree and write the prefix form or the postfix form.
   $$((A + B) - ((C * D) - (E/F))) * A$$

3. To display numbers (0 1234567890) we often use an electronic part called **the seven segment display** of the following shape:



   The LED bar corresponding to the value of variables a, b, c, d, e, f, g is lighted when the corresponding variable has value '1' and turned off when the variable has value '0'. For example $3 = 0011_2 = (x_4 x_3 x_2 x_1)_2$ is represented by $abcdefg = 1111001$. Let n be an integer satisfying $0 \leq n \leq 9$ (if $n \geq 10$, we don't care) and
   $$x_1 = n \bmod 2, x_2 = \lfloor n/2 \rfloor \bmod 2, x_3 = \lfloor n/4 \rfloor \bmod 2, x_4 = \lfloor n/8 \rfloor.$$

   (a) **(5pts)** Find the conjunctive normal form of a as a Boolean function of x_4, x_3, x_2, x_1.

   (b) **(5pts)** Find the disjunctive normal form of e as a Boolean function of x_4, x_3, x_2, x_1.

   (Hint for (a) and (b) : Draw tables of a and f versus binary representations of $0 \leq n \leq 9$ and consider a and e as functions of x_4, x_3, x_2, x_1. A conjunctive normal form is a conjunction of maxterms and a disjunctive normal form is a disjunction of minterms.)

Good Luck!