Applied Linear algebra

1. Let $V = \mathbf{R}^3$ and let (a_1, a_2, a_3) be a fixed vector. Show that the collection of elements (x_1, x_2, x_3) of V with $a_1x_1 + a_2x_2 + a_3x_3 = 0$ is a subspace of V. Determine the dimension of this subspace and find a basis.

2. Find all the eigenvalues of the following 3×3 matrix

$$\begin{pmatrix} 0 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

3. Let ϕ be a linear transformation from the finite dimensional $\mathbf{R}$-vector space V to itself such that $\phi \circ \phi = \phi$. Prove that $Im(\phi) \cap Ker(\phi) = 0$, where $Im(\phi)$ is the image of ϕ and $Ker(\phi)$ is the kernel of ϕ.