GRADUATION EXAM, ANALYSIS, SPRING 2015

1. Prove or disprove: the sequence of functions $f_n(x) = x^n$ converges uniformly on the interval $[0, 1)$.

2. Suppose that $f : [0, 2] \to \mathbb{R}$ is bounded on $[0, 2]$ and continuous on $[0, 2)$. Use the definition of Riemann-integrability to show that f is Riemann-integrable on $[0, 2]$.

3. (a) Let $f : X \to \mathbb{R}$ be a uniformly continuous function on the metric space X with metric d. If $\{a_n\}$ is a Cauchy sequence in X, show that the sequence $\{f(a_n)\}$ is a Cauchy sequence in $\mathbb{R}$.

(b) Give a counterexample to show that the conclusion may fail if f is merely continuous in (a).