Algebra II Qualifying Exam

June, 2015

1. (10 pts) For a field extension F/K with $[F : K]$ being prime, show that there are no infinitely many intermediate fields between F and K.

2. (a) (5 pts) Define a simple field extension F/K.

   (b) (15 pts) Find an element of the field $\mathbb{Q}(i, \sqrt{5})$ making it a simple extension over $\mathbb{Q}$.

3. (20 pts) Show that every finite Galois extension of $\mathbb{C}$ is Galois over $\mathbb{R}$.

4. (20 pts) Let F be a splitting field for $x^4 - 3$ over $\mathbb{Q}(i)$. Find the Galois group of F over $\mathbb{Q}(i)$.

5. (15 pts) For a PID R, prove or disprove that the polynomial ring $R[x]$ is a PID.

6. (15 pts) Consider a commutative ring R with a unique prime ideal P. Prove that $r \in R$ is nilpotent if and only if $r \in P$.