

1. Let  $X, Y$  be two random variables with joint distribution function  $F(x, y)$ . Verify that

$$\max(F_1(x) + F_2(y) - 1, 0) \leq F(x, y) \leq \min(F_1(x), F_2(y))$$

for all  $x \in \mathfrak{R}, y \in \mathfrak{R}$  where  $F_1(x)$  and  $F_2(y)$  are marginal distribution functions of  $X$  and  $Y$ , respectively.

2. Consider a disc with radius  $R$  where  $R$  is an exponential random variable with density

$$f(r) = e^{-r}, \quad 0 < r < \infty.$$

Find the distribution function of the area of the disc. What is the expected value of the area?

3. A random sample of size 16 from a normal distribution  $N(\mu, \sigma^2)$  yielded the sample mean of 1.5 and sample variance 9. Test whether  $H_0 : \mu \leq 0$  against  $H_1 : \mu > 0$  at 5% significance level. (You may use tables on the next page.)

TABLE A1 Standard Normal Distribution Function:  $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} dt$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
10	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
11	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
12	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
13	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
14	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
15	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
16	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
17	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
18	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
19	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
20	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
21	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
22	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
23	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
24	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
25	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
26	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
27	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
28	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
29	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
30	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
31	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
32	.9993	.9993	.9994	.9994	.9994	.9994	.9995	.9995	.9995	.9995
33	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
34	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

TABLE A3 Values of  $\alpha_n$

n	$\alpha = .10$	$\alpha = .05$	$\alpha = .025$	$\alpha = .01$	$\alpha = .005$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.474	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
$\infty$	1.282	1.645	1.960	2.326	2.576