

2014 Graduation Exam

November 20, 2014

1. Consider the mapping given by

$$F(x_1, x_2, x_3) = (x_1^2, x_2x_3, x_1x_2).$$

- (a) Prove this defines a surface.
- (b) Compute $(F_*)(\vec{v}_p)$ for the tangent vector \vec{v}_p with $\vec{v} = (1, 2, -1)$, $p = (1, 1, 1)$.

2. Consider the curve

$$\beta(s) = \left(\frac{(1+s)^{\frac{3}{2}}}{3}, \frac{(1-s)^{\frac{3}{2}}}{3}, \frac{s}{\sqrt{2}} \right)$$

which **has unit speed**. Compute its binormal B .

3. Consider the surface $z = xy$. Prove its Gaussian curvature is everywhere negative.