

Graduation examination, Applied Complex Variables

Fall 2014, Postech

1. (10 points) Use a contour integral to evaluate the improper integral

$$\int_0^{\infty} \frac{x^2}{4+x^4} dx.$$

2. (10 points) Let  $f$  be an analytic function in a nonempty open connected set  $G$ . Assume that  $|f(z)| = 2$  for every  $z \in G$ . Show that  $f$  is a constant function in  $G$ . (Hint. Consider the complex conjugate of  $f$ .)

3. (10 points) Let  $f$  be an analytic function in the unit disk  $D = \{z \in \mathbb{C} : |z| < 1\}$  with the power series expansion

$$f(z) = \sum_{n=0}^{\infty} a_n z^n, \quad z \in D.$$

Use the Cauchy integral formula to show that for  $n = 0, 1, 2, \dots$ , the coefficients  $a_n$  are given by

$$a_n = \frac{1}{2\pi r^n} \int_0^{2\pi} f(re^{i\theta}) e^{-in\theta} d\theta, \quad 0 < r < 1.$$