**Analysis**

November 2014

1. If $f$ is continuous on $[0,1]$ and if

$$\int_0^1 f(x)\, e^{nx}\, dx = 0 \qquad \text{for } n = 0, 1, 2, ...,$$

then using the Weierstrauss theorem, show that $f(x) = 0$ on $[0,1]$.

2. Let $f_n : [0;1] \to \mathbb{R}$ be a sequence of equicontinuous functions such that $f_n(x) \to f(x)$ for every $x \in [0,1]$. Prove that $f_n$ converges to $f$ uniformly.

3. Let $f_n \colon \mathbb{R} \to \mathbb{R}$ be a sequence of continuous functions such that

$$f_n \to f \quad \text{uniformly on } \mathbb{R}.$$

Given a sequence of points $x_n \in \mathbb{R}$ with $x_n \to x$, show that $f_n(x_n) \to f(x)$ as well.