ABSTRACT ALGEBRA 201405(GRADUATION)

Problem

(1) (10 pt) Show that any subgroup of a finite cyclic group must be cyclic again.

(2) (10 pt) Prove the following statement:

Let $\phi : G \longrightarrow H$ be a group homomorphism for groups G and F.
$ker(\phi) = \{e_G\}$ iff ϕ is injective.
Here $Ker(\phi) := \phi^{-1}(\{e_F\})$ and e_G and e_F are identites in G and F, respectively.

(3) (10 pt) Is $\mathbb{Z}_3 \times \mathbb{Z}_5$ a cyclic group? Justify your answer.

Remark Please note a passing grade ≥ 15.