

2014 NUMERICAL ANALYSIS EXAMINATION

Problem 1. Let f be a four times continuously differentiable function on $[-1, 1]$. Interpolate the function by a cubic polynomial $p_3(x)$ using the support points: $(-1, -3)$, $(0, 0)$, $(1, -2)$, $(2, 3)$.

Problem 2. From **Problem 1**, what is the maximal absolute interpolation error

$$\max_{x \in [-1, 1]} |f(x) - p_3(x)|.$$

Make the table for the divided differences at the nodes.

Problem 3. Consider the integral

$$\int_0^1 f(x) \, dx.$$

- (1) Derive the Simpson's rule for the integral.
- (2) Give the error representation for the quadrature rule.
- (3) Compute the error for the function $f(x) = x^3$ on $[0, 1]$.