MATH 261 Discrete Mathematics
Graduation Exam (May 2014)

1. Prove that the number of ways to choose r elements from $\{1, 2, \ldots, n\}$ allowing repetitions is equal to $C(n + r - 1, r)$.

2. Let $X = \{1, 2, \ldots, n\}$ and S be the set of one-to-one onto functions from X to itself.

   (a) Determine the cardinality of S.

   (b) Let $f \in S$. Show that there exist positive integers $\ell > m$ such that $f^{\ell}(x) = f^{m}(x)$ for all $x \in X$. Here, f^k denotes the composition

   $$\underbrace{f \circ f \circ \cdots \circ f}_{k}.$$

3. Let G be the graph whose adjacency matrix is

   $$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

   (a) Determine whether G has an Euler cycle.

   (b) Determine whether G is planar.

   (c) Suppose that the edges of G are given weights according to

   $$\begin{pmatrix} 0 & 1 & 2 & 1 & 0 \\ 1 & 0 & 0 & 2 & 3 \\ 2 & 0 & 0 & 0 & 2 \\ 1 & 2 & 0 & 0 & 1 \\ 0 & 3 & 2 & 1 & 0 \end{pmatrix}.$$

   (Recall that this means that the weight on the edge $\{i, j\}$ is the (i, j)-entry of the above matrix.) Find a minimal spanning tree.