Analysis
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1. A real-valued function f defined in (a, b) is said to be convex if $f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y)$ whenever $a < x < b, a < y < b, 0 < \lambda < 1$. Prove that every convex function is continuous.

2. If f is continuous on $[0, 1]$ and if

$$\int_0^1 f(x)\, x^n \, dx = 0 \qquad \text{for } n = 0, 1, 2, ...,$$

then using the Weierstrauss theorem, show that $f(x) = 0$ on $[0, 1]$.

3. Let $f_n : [a, b] \to \mathbb{R}$ be uniformly bounded continuous functions. Set

$$F_n(x) = \int_0^x f_n(t) dt, \quad a \leq x \leq b.$$

Prove F_n has a uniformly convergent subsequence.