



## Five Gold Button Math Competition

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Let  $N$  be a free abelian group of finite rank. Let  $u_1, \dots, u_n$  be elements of  $N$  and let  $N_i$  be the subgroup generated by  $u_1, \dots, u_i$ , with  $N_0 = 0$ . Suppose that for any  $j \geq 1$ ,  $\frac{1}{k} \bar{u}_j \notin N/N_{j-1}$  for any  $k > 1$ . Show that  $\{u_1, \dots, u_n\}$  can be expanded to a basis of  $N$ .



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