

2013 Fall 졸업시험 현대대수학 문제

[1] Let  $F$  be a finite field and  $n=|F|$ .

Prove that every element of  $F$  is a zero of the polynomial  $f(x)=x^n - x$ .

[2] Let  $\zeta \neq 1$  be a root of the equation  $x^3 - 1 = 0$ . Prove that  $Q(\zeta) \cong Q(\zeta^2)$  as fields, where  $Q$  is the field of rational numbers.

[3] Prove that every irreducible polynomial over  $R$  has degree 1 or 2, where  $R$  is the field of real numbers.