

### Analysis-Graduation Examination, 2013

1. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that  $f(0) = 0$  and  $|f(x) - f(y)| \leq 2|x - y|^{3/2}$  for all  $x, y \in \mathbb{R}$  and  $f(0) = 1$ .
2. Explain why any sequence  $\{a_n\}$  in  $[0, 1]$  has a convergent subsequence.
3. Let a function  $f : \mathbb{R} \rightarrow \mathbb{R}$  be continuous on  $\mathbb{R}$  and  $\{a_n\}$  be a Cauchy sequence in  $\mathbb{R}$ . Prove that  $\{f(a_n)\}$  is also a Cauchy sequence in  $\mathbb{R}$ .