

1. (40) Let X be a random variable with probability distribution

$$f(x) = pq^x, \quad x = 0, 1, 2, \dots, \quad 0 < p < 1, \quad q = 1 - p.$$

- (a) Verify that $f(x)$ is indeed a probability distribution.
- (b) Obtain the distribution function $F(x)$ of X .
- (c) Obtain the moment generating function $m(t) = E(e^{tX})$.
- (d) Find the mean and variance of X .

2. (30) Let X_1 and X_2 be two independent random variables from an exponential distribution with density $f(x) = e^{-x}$ for $x > 0$. Find the probability $P(|X_1 - X_2| > 1)$.

3. (30) A random sample of size 25 from a normal distribution yielded the sample mean of 30 and sample standard deviation of 10. Find a 95% confidence interval for μ .

TABLE A3 Values of $t_{\alpha,n}$

n	$\alpha = .10$	$\alpha = .05$	$\alpha = .025$	$\alpha = .01$	$\alpha = .005$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.474	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
∞	1.282	1.645	1.960	2.326	2.576

Other t probabilities:

$P\{T_8 < 2.541\} = .9825$ $P\{T_8 < 2.7\} = .9864$ $P\{T_{11} < .7635\} = .77$ $P\{T_{11} < .934\} = .81$ $P\{T_{11} < 1.66\} = .94$ $P\{T_{12} < 2.8\} = .984$.