Qualifying Exam
Algebraic Topology
June 2013

(1) (20pts) Let $X = \mathbb{R}^3 \setminus \{(x, y, 0) \mid x^2 + y^2 = 1\}$. Compute the homology groups of X.

(2) (a) (10pts) Let X be a finite CW complex. Show that

$$\chi(X) = \sum_n (-1)^n \operatorname{rank} H_n(X). \qquad (\dagger)$$

(b) (10pts) Compute the celluar homology groups of the compact orientable surface X of genus g, and verify the identity ($\dagger$).

(3) (20pts) Show that S^n has a nonvanishing continuous vector field if and only if n is odd.

(4) (20pts) Let $f : S^n \to S^n$ be a map satisfying $f(x) = f(-x)$ for all x. Show that $\deg f$ is even.

(5) (20pts) Let X be the surface obtained by identifying edges of a hexagon as follows.

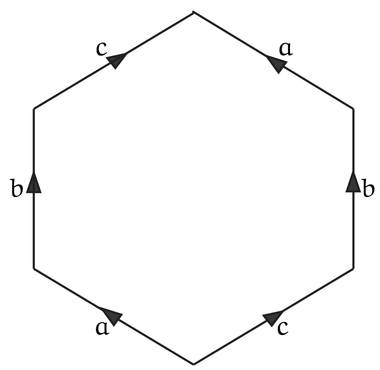

FIGURE 1

(a) (15pts) Compute the homology groups of X.
(b) (5pts) Construct a topological space Y which has the same homology groups as X but a different homotopy type.