1. Let the 3×3 matrix $A = \begin{bmatrix} 1 & 1 & \sqrt{2} \\ 0 & \sqrt{2} & 2 \\ 0 & 1 & \sqrt{2} \end{bmatrix}$ and solve the following problems.

   (a) Using the Gram-Schmidt process, find the QR factorization of the given matrix A, i.e., $A = QR$.

   (b) Using the QR factorization obtained in (a), find the least-squares solution of the equation $Ax = b$ when $b^T = [1, \sqrt{3}, \sqrt{3}]$.

2. Let the 2×2 matrix $A = \begin{bmatrix} 1 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ and answer the following questions.

   (a) Find the basis for the column space $C(A)$, the left null space $N(A^T)$, the row space $C(A^T)$ and the null space $N(A)$, respectively.

   (b) To get the least-squares solution of the equation $Ax = b$ when $b^T = [b_1,\ b_2]$, we firstly project the vector $b^T = [b_1,\ b_2]$ onto the $C(A)$, denoted by $\hat{b}$, and then find the inverse image of this vector in $C(A^T)$. That is, the least-squares solution $\hat{x}$ must satisfy the form $\hat{x} = \alpha \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ for some constant α since $[1,\ 1]^T \in C(A^T)$. Find the constant α and $\hat{b}$ in terms of $b^T = [b_1,\ b_2]$.

3. Let the 3×2 matrix $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 2 & 2 \end{bmatrix}$ and solve the following problems.

   (a) Find the SVD (Singular Value Decomposition) of the given matrix A, which has the form $U\Sigma V^T$, where U and V are orthonormal matrices and Σ is a diagonal matrix with singular values as its diagonal entries.

   (b) When the system of equation is given by $Ax = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 2 & 2 \end{bmatrix} x = b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, find the vector b_c which is the orthogonal projection of b onto the column space of A, denoted by $\mathbf{C}(A)$, using the projection matrix via orthonormal basis which spans $\mathbf{C}(A)$.