

Analysis-Graduation Examination, 2013

1. Determine whether $f(x) = \sqrt{x}$ is uniformly continuous on $(0, \infty)$ or not (Explain your reason).

2. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous and if $\{a_n\}$ is a Cauchy real sequence, determine whether $\{f(a_n)\}$ is also a Cauchy sequence or not (Explain your reason).

3. Let $f : [0, 1] \rightarrow \mathbb{R}$ is defined as follows:
 $f(x) = 0$ if x is a rational number and $f(x) = 1$ if x is an irrational number.

Determine whether f is Riemann-integrable on $[0, 1]$ or not (Explain your reason).