

1. (30) Let X be a nonnegative random variable with mean 1. Show that for any $k > 0$,

$$P(X \geq k) \leq \frac{1}{k}.$$

2. (40) Let X_1 and X_2 be two independent unbiased estimators for θ with (known) variances 1 and 2, respectively. Consider a combined estimator of the form $Y_a = aX_1 + (1 - a)X_2$ for real a .

(a) (20) Show that Y_a is unbiased for θ .

(b) (20) Choose a value of a at which the variance of Y_a is the smallest.

3. (30) Suppose that a survey showed that 80 of 100 randomly chosen Postech students said that they are unsatisfied with the cafeteria meals. Find a 95% confidence interval for the true proportion of unsatisfied Postech students.

(For the standard normal random variable Z , the points z_α satisfying $P(Z > z_\alpha) = \alpha$ are (i) 1.645 for $\alpha = 0.05$, (ii) 1.96 for $\alpha = 0.025$, and (iii) 2.24 for $\alpha = 0.0125$.)