Numerical Analysis **November 2012**

(1) Consider the following table.

x_i	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1

(a) Find a polynomial $p_3(x)$ which interpolates the table by either the Lagrange or Newton method.

(b) Let $e_3(x) = \sin(x) - p_3(x)$ be the error function. Write down the formula of the error function $e_3(x)$ with some higher derivative.

(c) Find an upper bound c such that

$$|e_3(x)| \leq c \qquad \text{for every } x \in [0, \frac{\pi}{2}],$$

where c is a constant real number.

(2) Consider the following quadrature rule $Q(f)$ which approximates $\int_{-1}^{1} f(x)dx$:

$$Q(f) = \frac{5}{9}f\Big(-\sqrt{\frac{3}{5}}\Big) + \frac{8}{9}f(0) + \frac{5}{9}f\Big(\sqrt{\frac{3}{5}}\Big).$$

Find the degree of precision (or exactness) of the quadrature rule.

(3) By using Gaussian elimination, find the lower triangular matrix L and the upper triangular U such that

$$L \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix} = R,$$

where

$$\text{diag}(L) = (1, 1, 1, 1, 1).$$

The End