

Differential Geometry, Exam. 2012 Fall.

1. Given a unit speed curve $\alpha(s)$ on a surface \mathcal{S} , explain the geodesic curvature κ_g and the normal curvature κ_n of α in terms of the shape operator L at a point $\mathbf{p} = \alpha(s) \in \mathcal{S}$. What is a geodesic? Can you state at least 3 basic properties of a geodesic curve?

2. Let $\triangle ABC$ be a triangle on a surface \mathbf{S} of negative Gaussian curvature bounded by three geodesic segments. Prove that the sum of the interior angles is less than π .

3. Find the total Gaussian curvature $\iint_{\mathcal{S}} K dA$ of the torus \mathbb{T}^2 that can be covered by

$$\begin{aligned} \varphi(u, v) &= ((a + r \cos u) \cos v, (a + r \cos u) \sin v, r \sin u), \\ &\text{for } r < a, 0 < u < 2\pi, 0 < v < 2\pi. \end{aligned}$$