Qualifying Exam
Algebraic Topology
December 2012

(1) (15pts) Compute the homology of the space obtained from S^2 by identifying two distinct points.

(2) (20pts) Let $f : S^2 \to S^2$ be a continuous extension of

$$z^5 + 2z^2 + 1 : \mathbb{C} \to \mathbb{C}.$$

Here, we are regarding S^2 as the one-point compactification of $\mathbb{C}$. Compute the degree of f.

(3) (15pts) Let X be a closed surface of Euler characteristic χ with a simplicial complex structure. Let v, e, f denote the number of vertices, of edges and of faces, respectively.

(a) (10pts) Show that the following (in)equalities hold:

$$f = 2(v - \chi), \quad e = 3(v - \chi), \quad e \leq v(v-1)/2.$$

(b) (5pts) Exhibit a simplicial complex structure on the two dimensional torus with minimal v, e, χ.

(4) (a) (15pts) Let X be the space obtained from the orientable surface of genus two by removing two distinct points p_1, p_2. Compute the cellular homology groups of X.

(b) (15pts) Enlarge the holes p_i to open balls B_i for each i and let Y be the space obtained by identifying ∂B_i to a_i, where a_i is the curve in Figure 1. Compute the homology groups of Y.

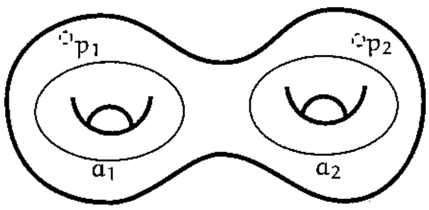

FIGURE 1

(5) (a) (10pts) Compute the integer cohomology rings of $S^2 \times S^2$ and $\mathbb{CP}^2$.

(b) (10pts) Prove that there is no odd degree map from $S^2 \times S^2$ to $\mathbb{CP}^2$.