

EXAM
Discrete Mathematics
November 2012

(1) Prove the identity

$$\binom{2n}{n} = \sum_{k=0}^n \binom{n}{k}^2.$$

(2) Let f_n denote the n th Fibonacci number. Show that the following holds:

$$\sum_{i=0}^{2n-1} f_i f_{i+1} = f_{2n}^2.$$

(3) True or False? You do not have to justify your answer.

- (T, F) The sum of the degrees of the vertices of a graph is always even.
- (T, F) If two vertices have the same degree, then there is a path between them without repeated vertices.
- (T, F) In any graph, there are even number of odd-degree vertices.
- (T, F) A complete graph has an Euler cycle.
- (T, F) If there is a unique path between any two distinct vertices, then the graph is a tree.
- (T, F) For any two distinct vertices of a tree, there is a unique path between them.
- (T, F) If a graph has v vertices and e edges and $v = e + 1$, then it is a tree.