

Calculus I

1. Calculate the integral

$$\int_0^1 x^3 \sqrt{1-x^2} dx.$$

2. Find the largest and smallest values of

$$f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$$

on the closed triangular plate bounded by the lines $x = 0$, $y = 2$ and $y = 2x$ on the first quadrant.

3. Calculate the flux

$$\iint_S \nabla \times \mathbf{F} \cdot \mathbf{n} d\sigma$$

of the vector field

$$\mathbf{F} = (x - y)\mathbf{i} + (y - z)\mathbf{j} + (z - x)\mathbf{k}$$

across the surface

$$S : \mathbf{r}(r, \theta) = \langle r \cos \theta, r \sin \theta, r \rangle, \quad \text{for } 0 \leq r \leq 5, \quad 0 \leq \theta \leq 2\pi$$

in the direction of outward unit normal.