

Applied Linear Algebra, 2012

1. Write down a  $3 \times 3$  matrix  $M$  with the property that for any  $3 \times 3$  matrix  $A$ ,

$$MA$$

is the matrix obtained from  $A$  by

- (1) adding 3 times row 1 to row 2;  
and then  
(2) switching row 1 and row 3.

For this  $M$ , find  $M^{-1}$ .

2.

We say an  $n \times n$  matrix  $A$  is *diagonalizable* if there is an invertible  $n \times n$  matrix  $S$  such that

$$S^{-1}AS$$

is diagonal.

- (a) Prove or disprove:  $A$  is diagonalizable if and only if it has  $n$  linearly independent eigenvectors.  
(b) Prove or disprove:  $A$  is diagonalizable if and only if it has  $n$  distinct eigenvalues.

3. Let  $A$  be a  $2 \times 2$  matrix with Jordan canonical form

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

Find the Jordan canonical form of  $A^{100}$ .