1. Let $f(z) = u(x,y) + iv(x,y)$ and be analytic in a domain D. Then prove the following problems.

   (a) Let $u(x,y) = C_1$ and $v(x,y) = C_2$, where C_1 and C_2 are constants. Show that these level curves are orthogonal at every point where $f'(z)$ is not zero.

   (b) If the function $f(z) = u(x,y) + iv(x,y)$ is has a constant real part, a constant imaginary part or a constant modulus, prove that the function itself is constant in each case.

2. Let C be a contour that avoids α and consider a contour integral

$$\int_C \frac{dz}{z-\alpha}.$$

   Further, assume that C is an arc $z = \zeta(t)$ for $a \le t \le b$ and define

$$L(t) = \int_a^t \frac{\zeta'(s)}{\zeta(s)-\alpha}\, ds \ (a \le t \le b).$$

   (a) Show that the function $J(t) = \exp[-L(t)][\zeta(t) - \alpha]$ is constant and determine that constant.

   (b) Show that if the contour C is closed, then the value of the following integral is the integer multiple of $2\pi i$.

$$\int_C \frac{dz}{z-\alpha}$$

3. Let $P(z)$ be a polynomial of degree $n\,(n \ge 1)$ with $a_n \ne 0$ and have zeros $z_k\,(k = 1, 2, \cdots, z_n)$, not necessarily distinct. Further, assume that $P(z)$ can be expressed in the form $P(z) = c(z - z_1)(z - z_2)\cdots(z - z_n)$, where c and $z_k\,(k = 1, 2, \cdots, z_n)$ are complex constants.

   (a) Suppose that $P(z)$ is a polynomial none of whose roots lie on a positively oriented simple closed contour C, then show that

$$\frac{1}{2\pi i}\int_C \frac{P'(z)}{P(z)}\, dz = \text{number of roots of } P(z) \text{ inside } C,$$

   where the roots are counted according to their multiplicity.

(b) If all the zeros of the polynomial $P(z)$ have negative real parts, prove that all the zeros of $P'(z)$ also have negative real parts.