수학과 졸업시험 (확률 및 통계) 2012년 1학기

1. (20) Let $P(A) = a$ and $P(B) = b$. Show that

$$\max(a + b - 1, 0) \le P(A \cap B) \le \min(a, b).$$

2. Let $X_1, X_2, \ldots, X_n$ be independent random variables from a distribution with mean μ and variance σ^2.

   (a) (20) Obtain the mean and variance of $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.

   (b) (40) Verify the identity

$$\sum_{i=1}^{n}(X_i - \mu)^2 = \sum_{i=1}^{n}(X_i - \bar{X})^2 + n(\bar{X} - \mu)^2.$$

   Use this identity to show that $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is an unbiased estimator for σ^2.

3. (20) A random sample of size 25 from a normal distribution with mean μ and variance 16 yielded the sample mean of 12. Find a 95% confidence interval for μ.
(For the standard normal random variable Z, the right tail cut-off points satisfying $P(Z > z_\alpha) = \alpha$ are $z_{0.05} = 1.645$, $z_{0.025} = 1.96$, $z_{0.0125} = 2.24$.)